ABSTRACTS

Post-Pareto approach in Multiobjective Optimal Control of Parabolic Systems

Henri Bonnel

ERIM, University of New Caledonia, France

This talk deals with the optimization of a functional over a Pareto control set associated with a convex multiobjective optimal control problem in Hilbert spaces, namely parabolic system. This approach generalizes for multiobjective optimal control of the dynamical systems governed by PDEs some results obtained in the case of multiobjective optimal control for the systems governed by ODEs. Two examples will be presented. General optimality results will be given, and a special attention will be paid to the linear-quadratic multiobjective parabolic system.

Convex Analysis on Groups and Semigroups

Jonathan M. Borwein

University of Newcastle, NSW, Australia.

In this talk we will show how we can canonically define the notion of convexity on additive groups and semigroups - so that in a vector space we have the classical notion. It turns out that even in relatively simple groups, convex sets can look quite different than in the classical vector space setting. Nevertheless, for large classes of groups and semigroups, many known results from convex analysis still hold in this more general setting. We will also discuss some aspects of convex analysis in topological groups. In particular, we will discuss a group-theoretic version of the Krein-Milman Theorem, as well as the Minimax Theorem. See https://www.carma.newcastle.edu.au/jon/ (This is joint work with Ohad Giladi.)

Proper Efficiency and Proper Karush-Kuhn-Tucker Conditions for Smooth Multiobjective Optimization Problems

Regina S. Burachik *and M.M. Rizvi*

University of South Australia

Proper Karush-Kuhn-Tucker (PKKT) conditions are said to hold when all the multipliers of the objective functions are positive. In 2012, Burachik and Rizvi introduced a new regularity condition under which PKKT conditions hold at every Geoffrion-properly efficient point. In general, the set of Borwein properly-efficient points is larger than the set of Geoffrion-properly efficient points. Our aim is to extend the PKKT conditions to the larger set of Borwein-properly efficient points.

Finding Dubins Path That Involves Interpolation by Using Optimal Control Theory

Yalcin Kaya

University of South Australia

Dubins path is the shortest planar curve joining two points with prescribed tangents, with a prescribed bound on its curvature. Lester Eli Dubins reported his solution on this in 1957 using geometric arguments. The structure of the solution is elegantly simple: a selection of at most three pieces of circular arcs of maximum curvature (the prescribed bound) and straight lines are concatenated. Dubins path has since been extensively used for path planning of unmanned aerial vehicles (drones) and robots. It has also been used for tunnelling in underground mines, where it is paramount to minimize the amount of earth excavated in opening a tunnel between two specified points and keep the tunnels short for ease of operations. In 1992, the original Dubins problem was transformed into an optimal control problem and the same solution was obtained by using optimal control theory. We consider a generalization of Dubins problem where the curve has to pass through intermediate points, a realistic situation in the practical settings mentioned above, giving rise to an interpolation problem. We transform the

interpolation problem into an optimal control problem and obtain a solution using optimal control theory.

Regularity of collections of sets: uniform and subregularity

Alexander Kruger

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Local regularity properties of finite collections of sets will be discussed with the main emphasis on uniform regularity and subregularity. Metric and dual quantitative characterizations of these properties as well as their relationships with the corresponding properties of set-valued mappings and illustrating examples will be presented.

SOCP Reformulation of the Generalized Trust Region Subproblem

Duan Li

The Chinese University of Hong Kong

We investigate in this research the generalized trust region subproblem (GTRS) of minimizing a general quadratic objective function subject to a general quadratic inequality constraint. By applying a simultaneous block diagonalization approach, we obtain a congruent canonical form for the symmetric matrices in both the objective and constraint functions. By exploiting the block separability of the canonical form, we show that all GTRSs with an optimal value bounded from below are second order cone programming (SOCP) representable. Compared to the state-of-the-art approach to reformulate GTRS as a semi-definite programming problem, our SOCP reformulation delivers a much faster solution algorithm.

Stability in linear optimization under total or partial data perturbations

Marco A. López Cerdá

University of Alicante, Spain

This talk deals with the stability of linear optimization problems with finitely many variables and an arbitrary number of constraints under total or partial perturbations of the coefficients. More in detail, we analyze the continuity properties of the feasible set, the optimal set and the optimal value, as well as the preservation of desirable properties (boundednesss, uniqueness) of the feasible and the optimal sets under sufficiently small perturbations.

On the Voronoi mapping

Juan Enrique Martínez-Legaz

Universitat Autonoma de Barcelona, Spain

Given an arbitrary closed set T in the Euclidean space whose elements are called sites and a particular site s; the Voronoi cell of s consists of all points closer to s than to any other site. The Voronoi mapping of s associates to each set T containing s the Voronoi cell of s with respect to T. The Voronoi cells are solution sets of linear inequality systems so they are closed convex sets. In this talk which is based on joint work with M. A. Goberna and V. N. Vera de Serio I will discuss the Voronoi inverse problem which consists in computing for a given closed convex set F containing a given site s those sets T that contain s and are such that the Voronoi cell of s with respect to T coincides with F. Set theoretical properties of the Voronoi mapping will also be discussed and explicit formulae for the inverse mapping will be provided.

Delayed Optimal Control Problems and Applications in Biomedicine

Helmut Maurer

University of Münster, Germany

There exists an extensive literature on delay differential models in biomedicine. However, only a few papers introduce optimal control techniques to determine optimal treatment programs on the basis of properly chosen objective functionals. We consider optimal control problems with control and state delays and briefly review the necessary conditions and discretization methods by which the control problem is transcribed into a large-scale nonlinear programming problem. First, we discuss the SEIR model in epidemiology [3] which describes the spread of an infectious disease. The model has two control variables, the rates of vaccination and treatment. The computed controls are a combination of bang-bang and singular arcs. The second model is a tuberculosis (TB) model [4] with time delays in the state and control variables. The controls represent the treatment rate and the chemotherapeutic dosis. For L^1–type objectives both controls are shown to be of bang-bang type. The third example concerns the treatment of cancer by a combination of an anti-angiogenic drug and a chemotoxic agent. The Gompertz type growth function considered in [1] typically leads to an anti-angiogenic control of bang-singular-bang type and a chemotoxic control of bang-bang type. For logistic type growth functions both controls are bang-bang [2].

References

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[3] H. Maurer and M.R. de Pinho, *Optimal control of epidemiological SEIR models with L^1–objectives and control–state constraints*, to appear in Pacific J. of Optimization, Special Issue devoted to the 70th birthday of Kok Lay Teo.

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Calculus of Variations on Unbounded Domains - a Hilbert Space Approach

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We consider a class of variational problems with infinite horizon with convex objective. The functional space for the state functions is assumed to be is a weighted Sobolev space.

This type of problems has different interesting applications.

On the one side it occurs by transformation of a singular variational problem - like the famous problem of Brachistochrone. On the other side, problems of economic growth can be formulated as such problems. But also the question of asymptotic controllabitity of an equilibrium state of a dynamic system appears in a new light if it is considered in weighted Sobolev spaces.

We present existence results and necessary optimality conditions for Pontryagin type minima in the considered spaces. A convex duality theory is developed which yields at the same time sufficient optimality conditions.

Using the results for the infinite horizon problem, we generate by backward transformation new theoretical results for singular variational problems over a finite horizon in a weighted space.

Solving Stochastic Variational Inequalities With Monotonicity by Progressive Hedging

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Stochastic variational inequality problems, single-stage or multistage, can be formulated with nonanticipativity as an explicit constraint on the interaction of decisions with information. The introduction of multipliers for that constraint allows in theory for a decomposition into separate subproblems each of the scenarios, as if hindsight were available, but a key question is how to generate the right value for those multipliers.

When monotonicity is present in the variational inequality, the progressive hedging algorithm of stochastic programming can be adapted to this setting. Solutions can then be calculated by iterations in which the multipliers, in parallel to those providing the present cost of future information in simpler models, are approximated and adjusted step by step.

Sparsity and Duality in Optimal Control

Christopher Schneider

University of Jena, Germany

We consider a general class of convex linear-quadratic (LQ) optimal control problems. Under certain assumptions on the problem data, the solutions of these problems are of *bang-bang type*. While it is well-known that employing L^2-regularization gives Lipschitz continuous optimal controls, incorporating L^1-control cost into the objective functional yields *sparse controls*, i.e. controls which are zero on whole intervals. The L^1-term can be seen as both a regularization parameter, since it promotes the sparsity of solutions, or a nonsmooth control cost, which models many practical applications in a more precise way than squared L^2-control costs. We will investigate the dependence of solutions on both the L^1- and the L^2-regularization parameter.

For the numerical solution of LQ problems with bang-bang solutions, we present an approach, which opts for solving the *dual problem* instead of the primal problem. For this purpose, we (i) construct the dual problem in the sense of Fenchel, (ii) prove that strong duality and a sattle point property hold, and (iii) prove convergence of the discretization.

Numerical experiments conclude the talk. We compare the numerical solution of the primal and dual problem, and figure out that significant computational savings can be obtained.

The semismooth Newton methods: 25 years in progress

Jie Sun

Curtin University
and
National University of Singapore

Newton's method is a classical numerical method for solving nonlinear equation systems. It is still the core of many modern algorithms in optimization. In 1993, I co-authored with L. Qi a paper to extend Newto's method to a class of nonsmooth equation systems called semismooth equation systems. This semismooth Newton method has soon become a major tool used for solving complementarity problems and variational inequalities. Since 2000, I have collaborated with D. Sun and others to further extend the semismooth Newton's method to matrix equations. Combined with the conjugate gradient method and the augmented Lagrangian method, this method has been shown to be particularly promising in solving huge-scale matrix optimization problems.

On set-valued optimization problems with variable ordering structure

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We introduce and investigate an optimality concept for set-valued optimization problems with variable ordering structure. In our approach, the ordering structure is governed by a set-valued map acting between the same spaces as the objective multifunction. Necessary optimality conditions for the proposed problem are derived in terms of Bouligand and Mordukhovich generalized differentiation objects.

Keywords: nondomination property · Pareto optimization · variable ordering structure · openness for sum-multifunction · necessary optimality conditions

Minimizing Control Variation in Nonlinear Optimal Control

Kok Lay Teo

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In any real system, changing the control signal from one value to another will usually cause wear and tear on the system's actuators. Thus, when designing a control law, it is important to consider not only predicted system performance, but also the cost associated with changing the control action. This cost, however, is almost always ignored in the optimal control literature. In this talk, we will consider a class of optimal control problems in which the variation of the control

signal is explicitly penalized in the cost function. We describe two computational methods - one based on a smooth approximation scheme and the other based on a novel transformation procedure - for solving this class of optimal control problems. We then apply these methods to solve example problems in fisheries, train control, and chemical engineering.

Optimization and Homotopy in Mathematical Handwriting Recognition

Stephen M. Watt

University of Waterloo, Canada

Mathematical handwriting, with its nested subscripts and superscripts, conveys information by the vertical placement of symbols. Unlike other areas of handwriting analysis, where a baseline can be used to disambiguate symbols, the opposite is true in mathematics—local vertical maxima, minima and inflections of symbols' curves determine various metric lines, including the baseline. We show how handwritten symbols can be modelled as parametric plane curves approximated by certain truncated orthogonal series. Symbols are then represented as points in a space of coefficients with dimension chosen to make symbol classes linearly separable. This allows linear homotopies between symbols of the same class, and in particular newly written symbols can be deformed smoothly to representative symbols with known points determining the metric lines. This can be used to find the neighbourhood of the corresponding determining points of the newly written symbol, after which numerical optimization finds the actual points.

Meta-optimisation: Lower bounds for higher faces

David Yost *and Guillermo Pineda and Julien Ugon*

Federation University Australia

As polytopes are the feasible regions of many optimisation problems, it is of interest to find bounds on the number of edges. We have previously announced such lower bounds for arbitrary d-dimensional polytopes with V vertices, when $V \leq 2d$, thereby confirming a 1967 conjecture of Grünbaum. We can now also give precise lower bounds for the number of m-dimensional faces, at least for $m \geq \frac{2}{3}d$. We present also the corresponding results for polytopes with $2d + 1$ vertices; in this case the minimum depends on whether or not d is prime.